-Asymptotic Methods in Mechanics of Thin-Walled Structures

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Part 1. Asymptotic methods in shell theory

Theory of shells contains many very complicated and ill-posed multi-parametric problems. This is a natural field for application of asymptotic approaches, because here researcher can find a lot of small (large) parameters. Let us list some of them:

- h/R ratio of shell thickness to its characteristic size, i.e. radius;
- l/L ratio of a period of non-homogeneity (l) to a characteristic size of considered structure (L) in the case of periodically non-homogeneous plates and shells;
- a/b ratio of characteristic sizes (plate length to its width);
- H/R ratio of shallow shell rise H to curvature radius R;
- w/h, where w is the normal displacement and h the plate thickness;
- B_1/B_2 ratio of bending stiffness of orthotropic shell;
- ω^{-1}, where ω is the dimensionless frequency of vibrations,

and so on.

Presence of above mentioned small parameters gives the possibility to use various asymptotic approaches: regular and singular perturbation, homogenization, artificial small parameter perturbation procedure, matched asymptotic expansions, etc.

In my talk I'll discuss problems of asymptotic simplification of isotropic shells boundary value problems and homogenization of periodically non-homogeneous plates and shells.

One of the most peculiar aspects of shell theory is associated with the existence of a few parameters of asymptotic integration yielding complexity of the problem being analyzed. In most studies this fact is omitted and therefore the domain of application results remains not clear. A.L.Gol'denveizer indicated the importance of estimating the order of coefficients of PDE and differential operators, he introduced the variation index of a function. I'll try to explain this technique using some simple, but important examples.

Part 2. Periodically non-homogeneous plates and shells

Analytical solutions will be derived. The asymptotic technique, based on homogenization method, will be used for this purpose. The homogenization method allows not only to obtain effective characteristics but also to investigate non-homogeneous distribution of mechanical stresses in different materials and structures what is of great significance for evaluating their strength.

Methods proposed are wide-ranged in applications and lead to simple and clear design formulae, useful for practical analyses. Successful examples will be demonstrated. In particular corrugated and perforated plates and shallow shells will be analysed.

The approach proposed will fill the substantial gap between numerical methods and approximate engineering approaches, based on heuristic hypotheses. Even if some numerical approach is used, shell analyst should remember, that "Design of computational or experimental schemes without the guidance of asymptotic information is wasteful at best, dangerous at worst, because of possible failure to identify crucial features of the process and their localization in coordinate and parameter space. Moreover, all experience suggests that asymptotic solutions are useful numerically far beyond their nominal range of validity, and can often be used directly, at least at a preliminary product design stage". (D.G.Crighton).