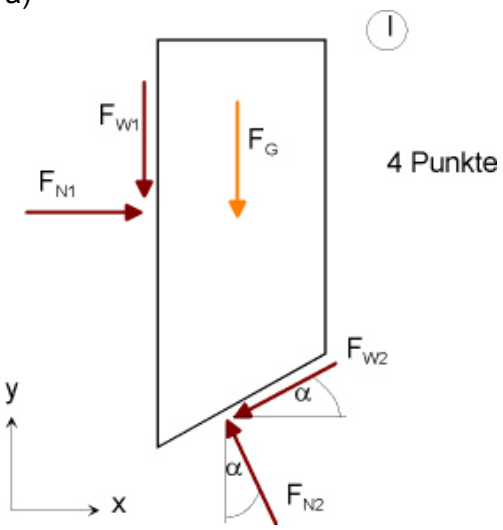
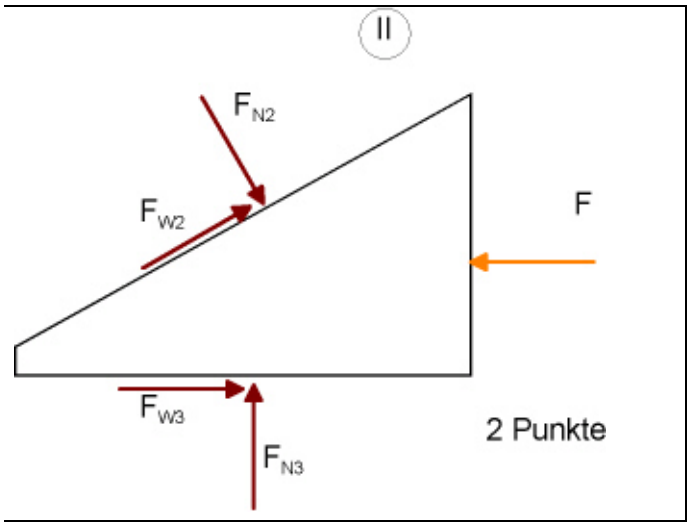
Klausur SS1997

Aufgabe 1

a)



4 Punkte



2 Punkte

b) System I:

$$\sum F_x = 0 : -F_{N2} \sin\alpha - F_{w2} \cos\alpha + F_{N1} = 0 \qquad (1)$$

$$\sum F_y = 0 : F_{N2} \cos\alpha - F_{w2} \sin\alpha - F_{W1} - F_G = 0 \qquad (2)$$

System II:

$$\sum F_x = 0 : F_{W2} \cos\alpha + F_{N2} \sin\alpha + F_{W3} - F = 0 \qquad (3)$$

$$\sum F_y = 0 : F_{W2} \sin\alpha - F_{N2} \cos\alpha + F_{N3} = 0 \qquad (4)$$

7 Punkte

$$F_{W1} = \mu F_{N1}, F_{W2} = \mu F_{N2}, F_{W3} = \mu F_{N3} \qquad (5.7)$$

Unbekannte: $F_{N1}, F_{W1}, F_{N2}, F_{W2}, F_{N3}, F_{W3}, F \Rightarrow 7$ Stück

(3) $\Rightarrow \quad F = F_{N2}(\mu \cos\alpha + \sin\alpha) + \mu F_{N3}$

(1) $\Rightarrow \quad -F_{N2}(\sin\alpha + \mu \cos\alpha) + F_{N1} = 0$

(2) $\Rightarrow \quad F_{N2}(\cos\alpha + \mu \sin\alpha) - \mu F_{N1} - F_G = 0$

(4) $\Rightarrow \quad F_{N2}(\mu \sin\alpha - \cos\alpha) + F_{N3} = 0$

aus (1) $\Rightarrow F_{N1} = F_{N2}(\sin\alpha + \mu \cos\alpha)$

aus (1)+(2) $\Rightarrow F_{N2}(\cos\alpha - \mu \sin\alpha - \mu \sin\alpha - \mu^2 \cos\alpha) = F_G$

aus (4) $\Rightarrow F_{N2} = \dfrac{F_G}{(\cos\alpha - 2\mu \sin\alpha - \mu^2 \cos\alpha)} \rightarrow F_{N3} = -\dfrac{\mu \sin\alpha \cos\alpha}{\cos\alpha - 2\mu \sin\alpha - \mu^2 \cos\alpha} F_G$

3 Punkte

$$\Rightarrow F = \frac{\mu \cos\alpha + \sin\alpha - \mu^2 \sin\alpha + \mu \cos\alpha}{\cos\alpha - 2\,\mu \sin\alpha - \mu^2 \cos\alpha} F_G = \frac{(1-\mu^2)\sin\alpha + 2\,\mu \cos\alpha}{(1-\mu^2)\cos\alpha - 2\,\mu \sin\alpha} F_G$$

c) $F \to \infty$: Selbstsperrung $\Rightarrow (1-\mu^2)\cos\alpha - 2\,\mu \sin\alpha = 0$

1 Punkt 1 Punkt

$$\Rightarrow \mu^2 + 2\,\mu \tan\alpha - 1 = 0$$

$$\Rightarrow \mu_{1,2} = -\tan\alpha \pm \sqrt{\underbrace{\tan^2\alpha + 1}_{\frac{1}{\cos^2\alpha}}}$$

$$= -\tan\alpha \pm \frac{1}{\cos\alpha}$$

2 Punkte

Insgesamt 20 Punkte

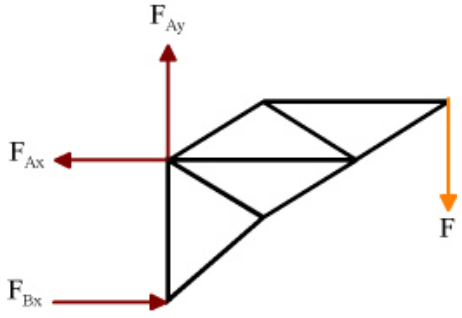
Aufgabe 2

a)

Anzahl der Lagerreaktionen: 3
Anzahl der Stäbe: 9
Anzahl der Knoten: 6

$\Rightarrow 2\,6 = 3 + 9$ statisch bestimmt!

b) Freischnitt:



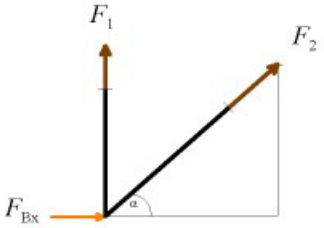
$$\sum F_x = 0 : F_{Ax} = F_{Bx}$$

$$\sum F_y = 0 : F_{Ay} = F$$

$$\sum M_{(B)} = 0 : -F\ 9a + F_{Bx}\ 4a = 0$$

$$\Rightarrow F_{Bx} = \frac{9}{4} F = \frac{9}{4} 4\ \text{kN} = 9\ \text{kN} = F_{Ax}$$

c) Stabkräfte 1 und 2:



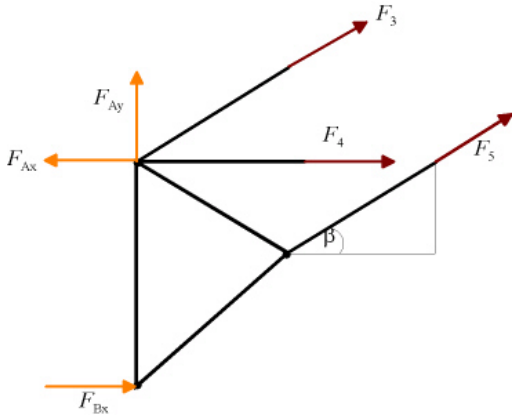
$$\sum F_x = 0 : F_{Bx} + F_2 \cos\alpha = 0$$

$$\alpha = 45°$$

$$\Rightarrow F_2 = \frac{-F_{Bx}}{\cos 45°} = \frac{-9\ \text{kN}}{\sqrt{2}} = -6.36\ \text{kN} \qquad \text{(Zug, Minus!)}$$

$$\sum F_y = 0 : F_1 = -F_2 \sin\alpha = +9\ \text{kN} \qquad \text{(Zug, Plus!)}$$

Stabkräfte 3 bis 5:



$$\sum M_{(A)} = 0 : +F_5 \cos\alpha\ a + F_5 \sin\beta\ 3a + F_{Bx}\ 4a = 0$$

$$F_5 = -\frac{4F_{Bx}}{(\cos\beta + 3\sin\beta)}$$

$$\sin\beta = \frac{1}{\sqrt{3^2 + 1^2}} = \frac{1}{\sqrt{10}}$$

$$\Rightarrow \beta = 18.43°$$

$$F_5 = -\frac{4\ 9\ \text{kN}}{(\cos 18.43° + 3\sin 18.43°)} = -18.98\ \text{kN}$$

$$\sum F_y = 0 : F_{Ay} + F_3 \sin\gamma + \text{F}\ \sin\gamma = 0, \quad \gamma = \beta$$

$$\Rightarrow F_3 = \frac{1}{\sin\gamma}\left(-F_{Ay} - F_5 \sin\gamma\right) = \frac{1}{\sin\gamma}\left(-4\ \text{kN} + 18.98\ \text{kN}\ \sin 18.43°\right) = 6.328\ \text{kN}$$

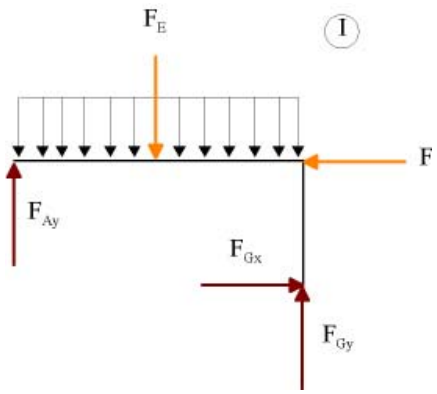
$$\sum F_x = 0 : -F_{Ax} + F_{Bx} + F_4 + F_3 \cos\gamma + F_5 \cos\beta = 0$$

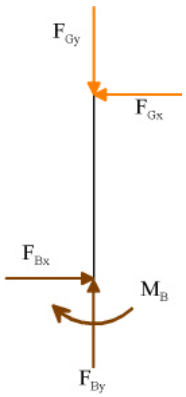
$$\Rightarrow F_4 = F_{Ax} - F_{Bx} - F_3 \cos\gamma - F_5 \cos\beta$$

$$= 9\ \text{kN} - 9\ \text{kN} - 6.328\ \text{kN}\ \cos 18.43° + 18.98\ \text{kN}\ \cos 18.43° = 12\ \text{kN}$$

Aufgabe 3

Freischnitt:





System I)

$$\sum F_x = 0 : F_{Gx} = F$$
$$\sum F_y = 0 : F_{Ay} - F_E + F_{ay} = 0$$
$$\sum M_{(G)} = 0 : -F_{Ay}\ 2\ a + F\ a + F_E\ a = 0$$

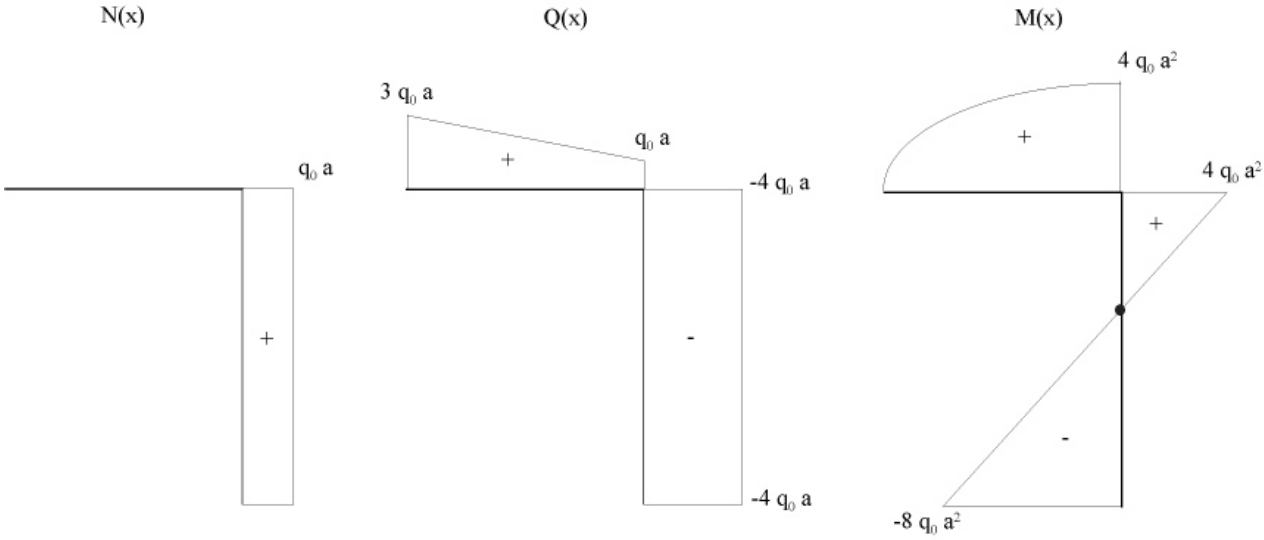
$$F_E = 2\ q_0\ a$$

$$\Rightarrow F_{Ay} = 3\ q_0\ a$$
$$\Rightarrow F_{Gy} = -q_0\ a$$
$$\Rightarrow F_{Gx} = 4\ q_0\ a$$

System II)

$$\sum F_x = 0 : -F_{Gx} + F_{Bx} = 0 \Rightarrow F_{Bx} = 4\ q_0\ a$$
$$\sum F_y = 0 : -F_{Gy} + F_{By} = 0 \Rightarrow F_{By} = -q_0\ a$$
$$\sum M_{(B)} = 0 : F_{Gx}\ 2\ a - M_B = 0 \Rightarrow M_B = 8\ q_0\ a^2$$

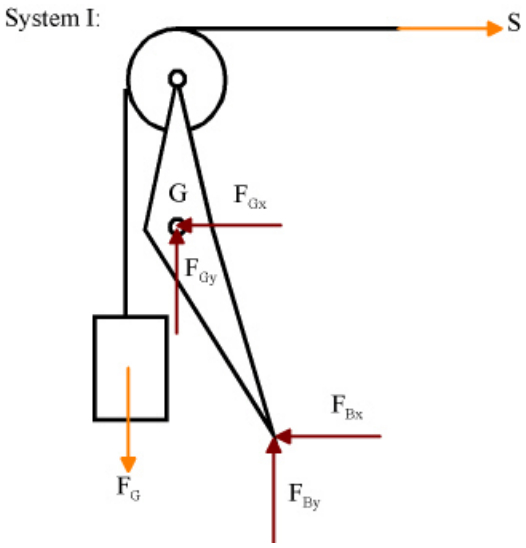
Verlauf der Schnittgrößen:

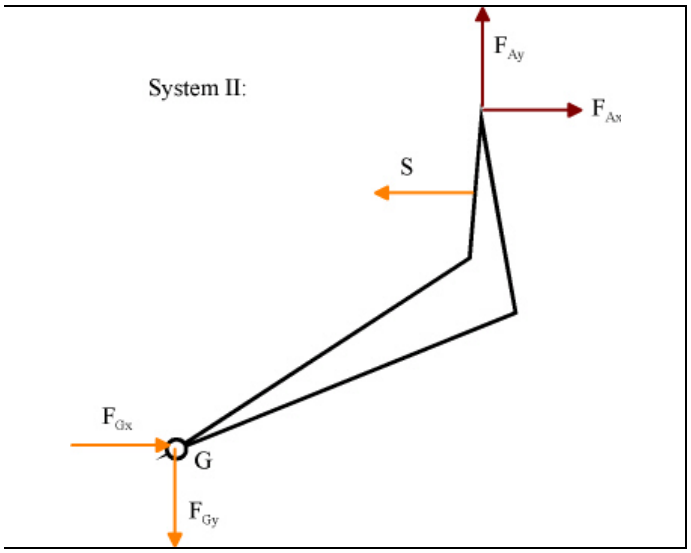


Aufgabe 4

a) Durch die Annahme, dass zwischen Seil und Rolle keine Reibungskräfte wirken, entspricht die Seilkraft der Gewichtskraft der am Seilende angebrachten Masse (=1 kN).

b) Freischnitt





System I)

$$\sum F_x = 0 : 1\,\text{kN} - F_{Gx} - F_{Bx} = 0$$
$$\sum F_y = 0 : -1\,\text{kN} + F_{Gy} + F_{By} = 0$$
$$\sum M_B = 0 : 3\,1\,\text{kN a -}\,1\,\text{kN}\,6\,a + F_{Gx}\,2\,a - F_{Gy}\,2\,a = 0$$

System II)

$$\sum F_x = 0 : 1\,\text{kN} + F_{Gx} + F_{Ax} = 0$$
$$\sum F_y = 0 : -F_{Gy} + F_{Ay} = 0$$
$$\sum F_A = 0 : F_{Gx}\,5\,a + F_{Gy}\,3\,a - 1\,\text{kN}\,a = 0$$
$$\Rightarrow F_{Gx} = F_{Gy} + \frac{3}{2}\,\text{kN}$$
$$\Rightarrow 5\,F_{Gx} + \frac{15}{2}\,\text{kN} + 3\,F_{Gy} - 1\,\text{kN} = 0$$
$$F_{Gy} = -\frac{13}{16}\,\text{kN}$$
$$F_{Gx} = \frac{11}{16}\,\text{kN}$$
$$F_{Bx} = \frac{5}{16}\,\text{kN}$$
$$F_{By} = \frac{29}{16}\,\text{kN}$$
$$F_{Ax} = \frac{5}{16}\,\text{kN}$$
$$F_{Ay} = -\frac{13}{16}\,\text{kN}$$

Aufgabe 5

a) $$\Delta l_{St} = \frac{l_{St}\ F}{E_{St}\ A_{St}}$$

$$\Delta l_{Cu} = \frac{l_{Cu}\ F}{E_{Cu}\ A_{Cu}} = \frac{l_{St}\ F}{\frac{E_{St}}{2}\ 2\ A_{St}}$$

$$2\ \Delta l_{St} + \Delta l_{Cu} = h$$

$$\Rightarrow \frac{3\ l_{St}\ F}{E_{St}\ A_{St}} = h \Rightarrow F = \frac{h\ E_{St}\ A_{St}}{3\ l_{St}} = \frac{1\,10^{-3}\ \text{m}\ 210\,10^{9}\ \frac{\text{N}}{\text{m}^2}\ 1\,10^{-4}\ \text{m}^2}{3\,1\,\text{m}\ 3} = 2.33\ \text{kN}$$

b) $$\sigma_{St} = \frac{F}{A_{St}} = \frac{2.33\ \text{kN}}{\frac{1}{3}\,10^{-4}\ \text{m}^2} = 69.9\ \text{MPa}$$

$$\sigma_{Cu} = \frac{F}{A_{Cu}} = \frac{2.33\ \text{kN}}{\frac{2}{3}\,10^{-4}\ \text{m}^2} = 35\ \text{MPa}$$

c) $$\Delta l_{St} = \alpha_{St}\ \Delta\upsilon\ l_{St}$$

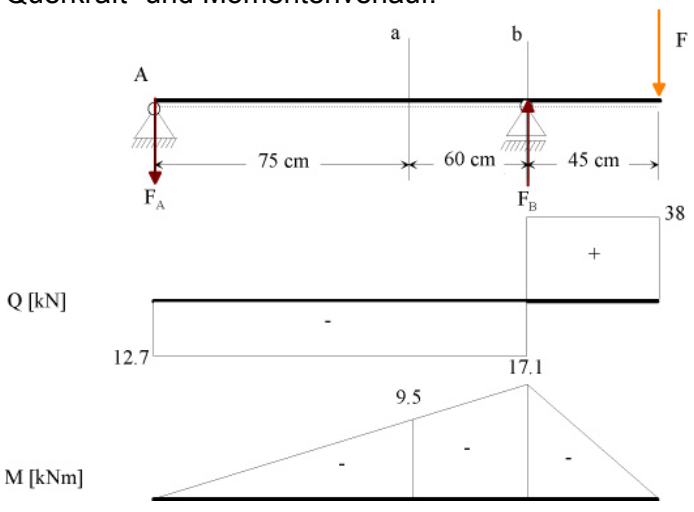
$$\Delta l_{Cu} = \alpha_{Cu}\ \Delta\upsilon\ l_{Cu}$$

$$\Rightarrow 2\ \Delta l_{St} + \Delta l_{Cu} = (2\ \alpha_{St}\ l_{St} + \alpha_{Cu}\ l_{Cu})\ \Delta\upsilon = h$$

$$\Rightarrow \Delta\upsilon = \frac{h}{2\ \alpha_{St}\ l_{St} + \alpha_{Cu}\ l_{Cu}} = \frac{10^{-3}\ \text{m}}{(2\,12\,1\,\text{m} + 1.16\ \text{m})\,10^{-6}\ \frac{1}{\text{K}}} = 25\ \text{K}$$

Aufgabe 6

Querkraft- und Momentenverlauf:



$$\sum \mathrm{M}_\mathrm{A} = 0 = F_B\ 135\ \mathrm{cm} - F\ 180\ \mathrm{cm}$$
$$\sum F_y = 0 = -F_A + F_B - F$$

mit $F = 38\ \mathrm{kN}$
$\Rightarrow F_B = 50.7\ \mathrm{kN}$
$\Rightarrow F_A = 12.7\ \mathrm{kN}$

	A_i	y_i	$A_i\ y_i$	$\lvert a\rvert = \lvert y_s - y_{si}\rvert$	$a^2\ A$	I_0
400 8	32.0	0.4	12.8	3.73	445	≈0
150 5	15.0	8.3	124.5	4.17	261	281
100 5	5.0	15.55	77.8	11.42	652	≈0

$$\sum A_i = 52.0\ \mathrm{cm}^2$$
$$\sum A_i\ y_i = 215\ \mathrm{cm}^3$$
$$y_s = \frac{\sum A_i\ y_i}{\sum A_i} = 4.13\ \mathrm{cm}$$

$$I_x = 1640\ \mathrm{cm}^4$$

Biegespannungen: $\sigma = \frac{M}{W} = \frac{M}{I}\,e$

$$\sigma_{oben} = \frac{9.50\ \mathrm{kN}\ 4.13\ \mathrm{cm}}{1640\ \mathrm{cm}^4} \Rightarrow \sigma_o = 23.9\ \frac{\mathrm{N}}{\mathrm{mm}^2}$$
$$\sigma_{unten} = \frac{-9.50\ \mathrm{kN}\ 11.67\ \mathrm{cm}}{1640\ \mathrm{cm}^4} \Rightarrow \sigma_u = -67.6\ \frac{\mathrm{N}}{\mathrm{mm}^2}$$

Schubspannungen: $\tau = \frac{F_Q \sum A_i\ y_i}{I_x\ 2a}$ $\qquad a = 3\ mm;\ F_Q = 12.7\ \mathrm{kN}$

$$\tau = \frac{12.7\ 10^{-3}\ \mathrm{N}\ 400\ 8\ \mathrm{mm}^3\ (41.3 - 4)\ \mathrm{mm}}{1640\ 10^4\ \mathrm{mm}^4\ 2\ 3\ \mathrm{mm}} \Rightarrow \tau = 15.4\ \frac{\mathrm{N}}{\mathrm{mm}^2}$$

$$\sigma_{Naht} = \frac{M}{I_x}(4.13 - 0.8)\mathrm{cm} = 19.3\ \frac{\mathrm{N}}{\mathrm{mm}^2}$$

Aufgabe 7

Steifigkeiten:

$EI_1 = 2000 \text{ kNm}$

$EI_2 = 1800 \text{ kNm}$

Wahre Auflager und Momente:

$$F_B = \frac{1}{6}\left[-8\ 0.5 + 23\ 0.5\right] = 1.25 \text{ kN}$$

$$F_{Ay} = -F_B = -1.25 \text{ kN}$$

$$M_{Ar} = -8 \text{ kN } 0.5 = -4 \text{ kNm}$$

$$M_{kl} = -8 \text{ kNm } 0.5 \text{ m} - 1.25 \text{ kN } 3 \text{ m} = \text{-}7.75 \text{ kNm}$$

$$M_{kr} = +1.25 \text{ kNm } 3 \text{ m} = +3.75 \text{ kNm}$$

Ersatzkräfte:

$$F_a = \frac{2.0 + 3.88}{2}\ 3.0\ 10^{-3} = 8.82\ 10^{-3}$$

$$F_b = \frac{1}{2}\ 2.08\ 3\ 10^{-3} = 3.12\ 10^{-3}$$

Ersatzauflager:

$$F_B = \frac{1}{6}\left[3.12\ 10^{-3}\ \ 4.0 - 8.82\ 10^{-3}\ 1.7\right] = -0.419\ 10^{-3}$$

$$F_A = \left(8.82 - 3.12 - 0.419\right)10^{-3} = 5.28\ 10^{-3}$$

Ersatzmomente:

$$M_a = -5.88\ 10^{-3}\ 1.7 \text{ m} = -8.98 \text{ mm}$$

$$M_b = -0.419\ 10^{-3}\ \ 2.0 \text{ m} = \text{-}0.84 \text{ mm}$$

Neigung und Durchbiegung:

$$\zeta_i = \frac{8.98 \text{ mm}}{1.7 \text{ m}} = 5.23\ 10^{-3}$$

$$y_k = -\left[0.84 + \frac{\left(8.98 - 0.84\right)}{2.3}\ 1.0\right] = -4.38 \text{ mm}$$