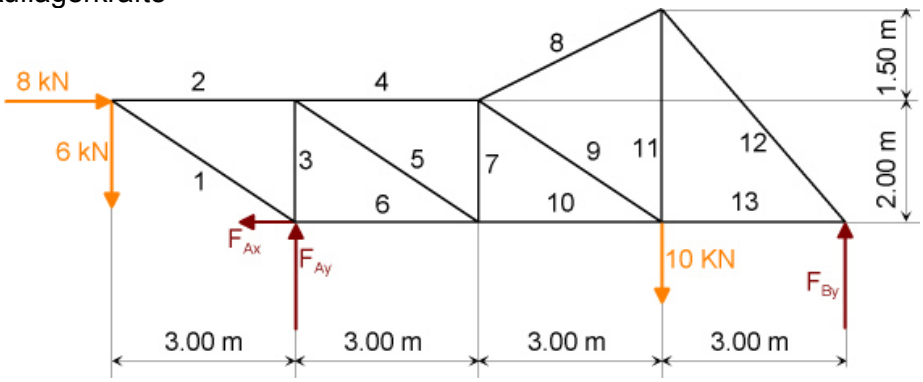
Klausur SS1996

Aufgabe 1

a) Ermittle die Auflagerkräfte



$$\sum F_A = 0 : 8\,\text{kN} - F_{Ax} = 0 \Rightarrow F_{Ax} = 8\,\text{kN}$$

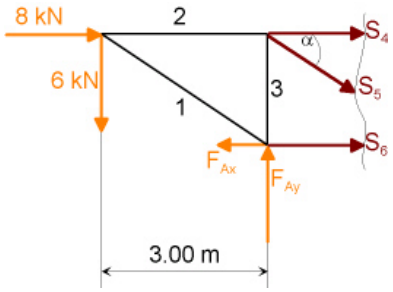
$$\sum F_y = 0 : -6\,\text{kN} + F_{Ay} - 10\,\text{kN} + F_{By} = 0 \Rightarrow F_{Ay} = 16\,\text{kN} - F_{By}$$

$$\sum M_{(A)} = 0 : -8\,\text{kN}\ 2\,\text{m} + 6\,\text{kN}\ 3\,\text{m} - 10\,\text{kN}\ 6\,\text{m} + F_{By}\ 9\,\text{m} = 0$$

$$\Rightarrow F_{By} = \frac{1}{9}(16 - 18 + 60)\,\text{kN} = \frac{58}{9}\,\text{kN} = 6.44\,\text{kN}$$

$$F_{Ay} = (16 - 6.44)\,\text{kN} = 9.56\,\text{kN}$$

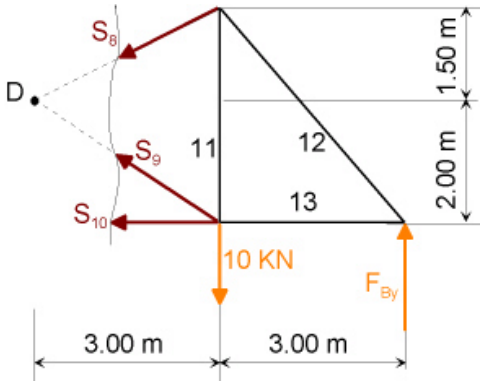
b) Ermittle Stabkraft S_5:



$$\sum F_y = 0 : -6\,\text{kN} + F_{Ay} - S_5 \sin\alpha = 0$$

$$\Rightarrow S_5 = \frac{(9.56 - 6)\,\text{kN}}{\sin\left(\arctan\frac{2}{3}\right)} = \frac{3.56\,\text{kN}}{\sin 33.69°} = +6.42\,\text{kN}$$ **Zugstab!**

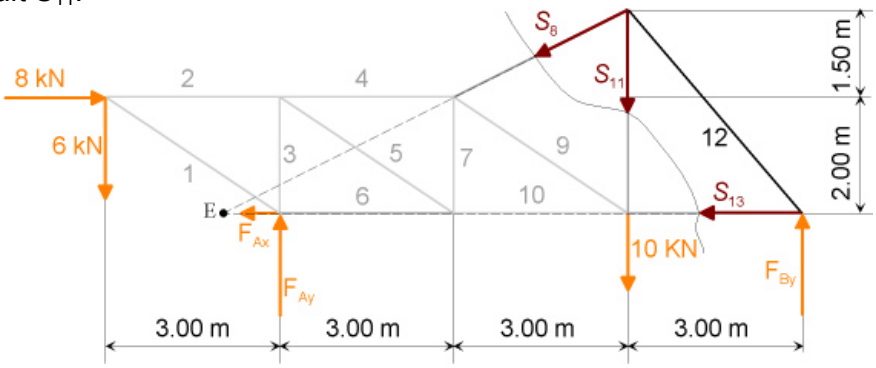
c) Ermittle Stabkraft S_{10}:



$$\sum M_{(D)} = 0 : F_{By}\ 6\ \text{m} - 10\ \text{kN}\ 3\ \text{m} - S_{10}\ 2\ \text{m} = 0$$

$$\Rightarrow S_{10} = \frac{+6.44\ 6 - 30}{2}\ \text{kN} = +4.32\ \text{kN} \qquad \textbf{Zugstab!}$$

d) Ermittle Stabkraft S_{11}:



$$F_B = \frac{1}{9\ \text{m}}\left(+8\ 2.0\ \text{m} - 6\ 3.0\ \text{m} + 10\ 6.0\ \text{m}\right)\text{kN} = 6.44\ \text{kN}$$

$$S_5 \sin\alpha = \left(10 - 6.44\right)\text{kN} \qquad \tan\alpha = \frac{2}{3} \qquad \alpha = 33.7°$$

$$S_5 = \frac{3.56}{\sin 33.7°}\ \text{kN} = +6.41\ \text{kN}$$

$$\sum M_D = 0 : S_{10}\ 2.0 + \left(10\ 3.0 - 6.44\ 6.0\right)\text{kN} = 0$$

$$S_{10} = \frac{1}{"}\left(6.44\ 6.0 - 10\ 3.0\right)\text{kN} = +4.37\ \text{kN}$$

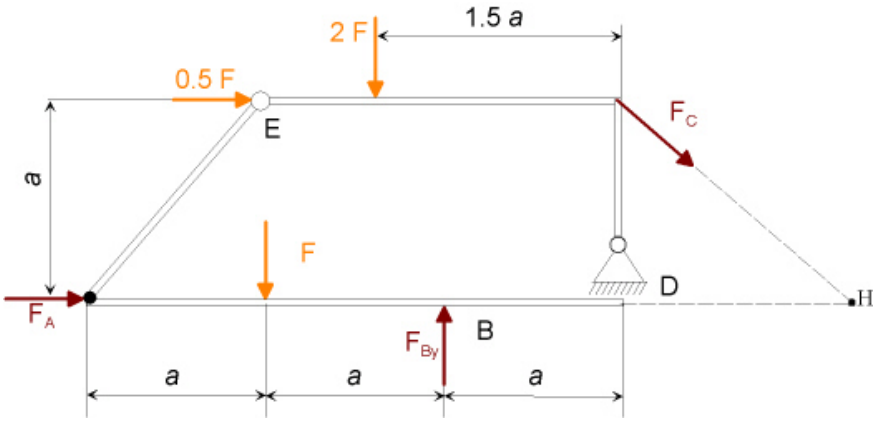
$$\sum M_E = 0 : S_{11}\ 7.0\ \text{m} - 6.44\ ´\text{kN}\ 10.0\ \text{m} = 0$$

$$S_{11} = +9.2\ \text{kN}$$

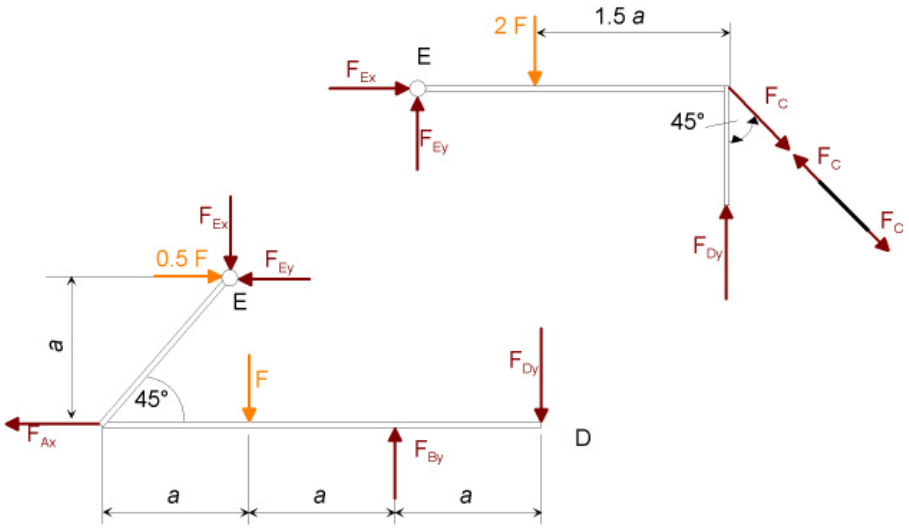
Aufgabe 2

Freischnitt

(I) Gesamtsystem:



(II) Zwei Teilsysteme:



Gleichungen (Scheibe I)

$$\sum F_x = 0 : F_{Ex} + \frac{F_c}{\sqrt{2}} = 0$$

$$\sum F_y = 0 : F_{Ey} - 2F - \frac{F_C}{\sqrt{2}} + F_{Dy} = 0$$

$$\sum M_{(C)} = 0 : 2F\ 1.5\ a - F_y\ 2\ a = 0$$

$$\Rightarrow F_{By} - 4.5\ F + \frac{F_C}{\sqrt{2}} = 0 \Rightarrow F_C = \sqrt{2}\ 0.75\ F = 1.06\ F \Rightarrow F_{Ex} = -0.75\ F$$

$$\Rightarrow F_{Dy} = 1.25\ F$$

$$\Rightarrow F_{By} = 1.5\ F$$

Gleichungen (Scheibe II)

$$\sum F_x = 0 : -F_{Ax} - F_{Ex} + 0.5\ F = 0$$

$$\sum F_y = 0 : -F_{Ey} + F_{By} - F_{Dy} - F = 0$$

$$\sum M_{(B)} = 0 : -F_{By}\ a + F\ 2a + F_{Ex}\ a + F_{Ex}\ a + \underbrace{F_{By}}_{1.5\ F}\ 2a = 0$$

$$\Rightarrow F_{Ax} = 1.25\ F$$

Gleichungen Gesamtsystem

$$\sum M_{(H)} = 0 : 2F\ 2.5a + F\ 3a - F_B\ 2a - 0.5F\ a = 0$$

$$\Rightarrow F_B = 3.75F$$

Gesamtsystem

$$\sum M_B = 0 : -F_C\ a\sqrt{2} - 2F\ 0.5a - F\ a + 0.5F\ a$$

$$F_C = \frac{F}{\sqrt{2}}\left[-1-1+0.5\right] = -\frac{1.5}{\sqrt{2}}F$$

$$\sum F_x = 0 : F_A + \frac{1.5}{2}F + 0.5F = 0$$

$$F_A = -1.25F$$

$$\sum F_y = 0 : F_B - F - 2F - \frac{1.5}{2}F = 0$$

$$F_B = +3.75F$$

Scheibe II

$$\sum M_E = 0 : -F_D\ 2a + 2F\ 0.5a + \frac{1.5}{2}F\ 2a = 0$$

$$F_D = \frac{F}{2}\left[1 + 0.75\ 2\right] = +1.25F$$

$$\sum F_x = 0 : +F_{Ex} + \frac{1.5}{2}F = 0$$

$$F_{Ex} = -0.75F$$

$$\sum F_y = 0 : +F_{Ey} - 2F + 1.25F - \frac{1.5}{2}F = 0$$

$$F_{Ey} = +1.5F$$

Aufgabe 3

a) Berechnung der Auflagerkräfte

$$\sum F_x = 0 : 4.5\ \text{kN} - F_{Ax} - F_B \sin 40° = 0$$
$$\sum F_y = 0 : F_{Ay} - 8\ \text{kN} + F_B \cos 40° = 0$$
$$\sum M_{(A)} = 0 : -4.5\ \text{kN}\ 0.8\ \text{m} - 8\ \text{kNm}\ 1\ \text{m} + F_B(\sin 40°\ 1.6\ \text{m} + \cos 40°\ 2\ \text{m}) = 0$$

$$\Rightarrow F_B = \frac{4.5\ 0.8 + 8\ 1}{\sin 40°\ 1.6 + \cos 40°\ 2}\ kN = 4.53\ \text{kN}$$
$$F_{Ax} = (+4.5 - 4.53 \sin 40°)\ \text{kN} = +1.59\ \text{kN}$$
$$F_{Ay} = (8 - 4.53 \cos 40°)\ \text{kN} = +4.52\ \text{kN}$$

b) Berechnung der Schnittgrößen

$$-F_B \sin 40° = -4.53 \sin 40° = -2.91$$

$$-F_B \cos 40° = -4.53 \cos 40° = -3.47$$

$$1.59\ 0.8 = 1.27\ \text{kNm}$$
$$1.59\ 1.6 - 4.5\ 0.8 = -1.06\ \text{kNm}$$

c) Berechnung von Ort und Größe des maximalen Biegemomentes im Bereich der Gleichstreckenlast

1. Suche der Nullstelle der Querkraft

$$\frac{4.52}{x} = \frac{3.47}{2 - x} \Rightarrow 2\ 4.52 - 4.52\ x = 3.47\ x \Rightarrow x = 1.13\ \text{m}$$

2. Gleichung der Parabel

$$M(x) = A + Bx + Cx^2$$
$$M(x = 0) = -1.06\ \text{kNm} = A$$
$$M(x = 2\ \text{m}) = 0 = A + 2\ B\ \text{m} + 4\ C\ \text{m}^2$$

$$\Rightarrow B = -2\ C\ \text{m} + 0.53\ \text{kN}$$

$$\left.\frac{dM}{dx}\right|_{x=1.13\ \text{m}} = 0 = B + 2\ C\ 1.13\ \text{m}$$

$$0 = -2\ C\ \text{m} + 2\ C\ 1.13\ \text{m} + 0.53\ \text{kN} = -2.0385$$
$$C = -\frac{0.53\ \frac{\text{kN}}{\text{m}}}{0.26} \qquad B = 4.607\ \text{kN}$$

$$\Rightarrow M(x = 1.13\ \text{m}) = -1.06\ \text{kNm} + 4.607\ \text{kNm}\ 1.13\ \text{m} - 2.0385\ \text{kN}\ 1.13^2\ \text{m}^2 = 1.54\ \text{kNm}$$

$$-F_B \sin 40° \, 1.6 - F_B \cos 40° \, 2.0 + (4.5 \, 0.8 + 4 \, 2 \, 1) \text{ kN} = 0$$

$$F_B = \frac{4.5 \, 0.8 + 4 \, 2 \, 1}{1.6 \sin 40° + 2.0 \cos 40°} \text{ kN} = \frac{11.6}{2.56} \text{ kN} = 4.53 \text{ kN}$$

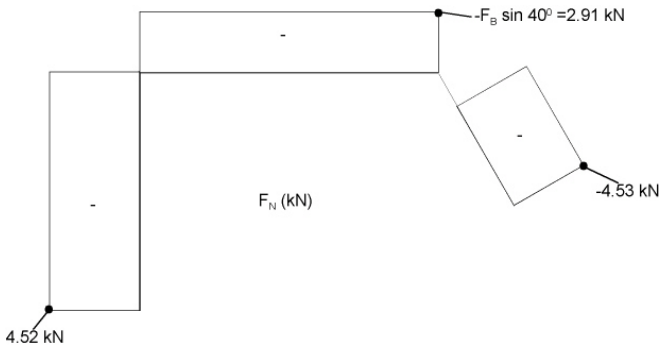
$$F_{Bx} = 4.53 \text{ kN} \sin 40° = 2.91 \text{ kN}$$
$$F_{By} = 4.53 \text{ kN} \cos 40° = 3.47 \text{ kN}$$

$$F_{Ax} = 4.5 - 2.91 = 1.59 \text{ kN}$$
$$F_{Ay} = 8.0 - 3.47 = 4.53 \text{ kN}$$

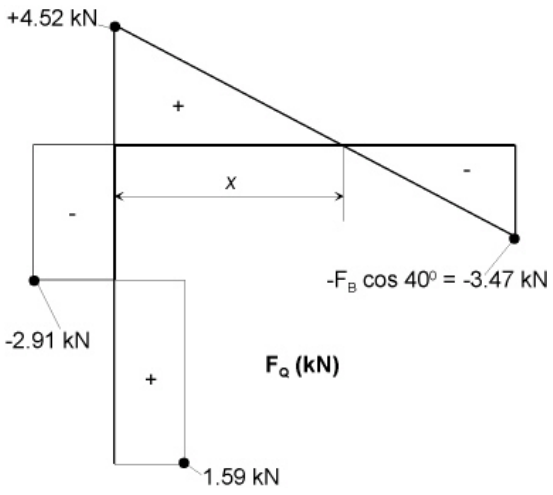
$$M_1 = +1.59 \, 0.8 = +1.27 \text{ kNm}$$
$$M_2 = +1.59 \, 1.6 \, 4.5 \, 0.8 = -1.06 \text{ kNm}$$
$$M_3 = +3.47 \, 1.0 = +3.47 \text{ kNm}$$
$$M_0 = \frac{4 \, 2^2}{8} = 2.0 \text{ kNm}$$

$$M_{\max} = 3.47 \, 0.87 - \frac{4 \, 0.87^2}{2} = 1.51 \text{ kNm}$$

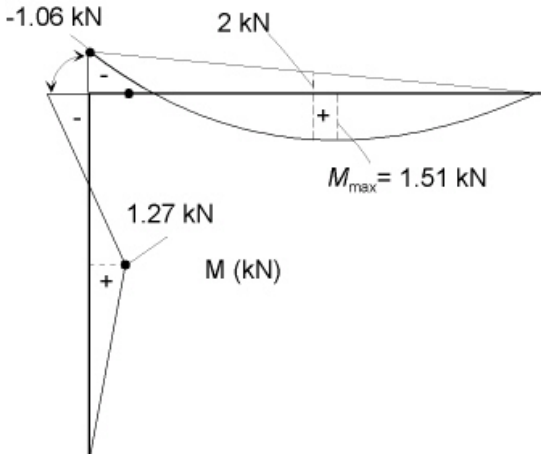
Normalkraftverlauf:



Querkraftverlauf:

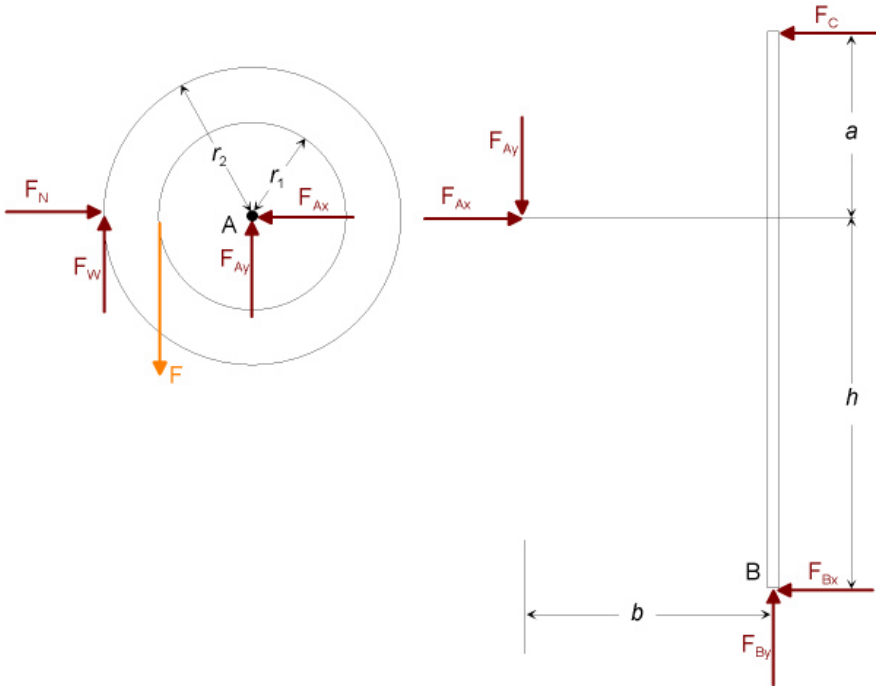


Momentenverlauf:



Aufgabe 4

a) Freischnitt:



$$F_W = \mu F_N$$

$$F_W r_2 = F r_1 \Rightarrow F_W = F \frac{r_1}{r_2}$$

$$-F_C(h+a) - F(b+r_1) + F_W(b+r_2) + F_N h = 0$$

$$-F_C(h+a) - F(b+r_1) + F\frac{r_1}{r_2}\left(b + r_2 + \frac{\mu}{h}\right) = 0$$

$$F_C = \frac{F}{h+a}\left[\frac{r_1 - r_2}{r_2} b + \frac{h\, r_1}{\mu\, r_2}\right]$$

b) Selbstsperrung:

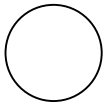
$$F_C = 0$$

$$b\,\frac{r_2 - r_1}{r_2} = \frac{h\,r_1}{\mu\,r_2}$$

$$\mu = \frac{h\,r_1}{b(r_2 - r_1)}$$

Aufgabe 5

$$I_o = \frac{\pi\,d^4}{64}$$

Teilkörper	A_i	y_i	$A_i\,y_i$	a	$A\,a^2$	I_o
	95.03	17.5	1663	5.67	3055	719
	-50.27	17.5	-880	5.67	-1616	-201
	29.6	3.28	97	8.55	2164	316
$\sum$	74.36		880		I_{xx} = 4437	

$$y_s = \frac{\sum A_i\,y_i}{\sum A_i} = \frac{880}{74.36} = 11.83\ \text{cm} \qquad W = \frac{I_{xx}}{l}$$

$$W_1 = \frac{4437\ \text{cm}^4}{(23 - 11.83)\ \text{cm}} = 397.2\ \text{cm}^3$$

$$W_2 = \frac{4437\ \text{cm}^4}{(12 - 11.83)\ \text{cm}} = 26100\ \text{cm}^3$$

$$W_3 = \frac{4437\ \text{cm}^4}{11.83\ \text{cm}} = 375\ \text{cm}^3$$